

Low Energy Description of Fermion Pairs in Topologically Massive QED_{2+1} with N Flavours

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ABSTRACT

In topologically massive QED_{2+1} with N flavours, there is the possibility that two equal-charged fermions can form a bound state pair in either s-wave or p-wave. We are concerned about the s-wave pairs and obtain the low energy effective action describing them. It is shown that the fermion pairs behave like doubly charged spin-1 bosons and, when they condense, the gauge field acquires the longitudinal mass. The approximate $SU(2)$ symmetry due to the similarity between the fermion pairs and the gauge field is discussed.

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The topologically massive QED in (2+1) dimensions [1] is the usual QED with a Chern-Simons (CS) term and its Lagrangian is of the form as follows:

$$\mathcal{L}_{\text{TMQ}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\kappa}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda - eA_\mu j^\mu + \mathcal{L}_m \quad (1)$$

where j^μ is the matter current and \mathcal{L}_m the kinetic term of matter. One of the features of this model is that, due to the structure of the CS term, a charged particle at rest is a source of not only an electric field but also a magnetic field. Based on this, it has been shown in ref. [2] that the magnetic moment leads to a *short-range* interaction between particles, and the particle dynamics is drastically changed. In the subsequent works [3,4], it has also been shown that bosonic as well as fermionic matter can have its magnetic moment.

For the fermionic particles, there appears an interesting possibility that the interaction due to the magnetic moment may be attractive even between equally charged fermions and hence it is expected that bound states appear under certain circumstances. Some authors have investigated on this possibility and argued that, although the exact analytical solution has not yet been given, it is indeed possible that such bound states appear [2,5–7].

The strength and the form of the interaction between two equally charged fermions mediated by topologically massive photon are obtained by calculating the amplitude \mathcal{M} of Möller scattering. In refs. [2,4–7], the tree level calculations of \mathcal{M} have been done in the nonrelativistic limit as follows: i) For the scattering of fermions with different types in an antisymmetric manner, the amplitude, say \mathcal{M}_s , gives a pure s-wave interaction $\mathcal{M}_s = -(e^2/m_T^2)(m_T/m - 1)$ where $m_T (= \kappa/2\pi)$ is the topological (transverse) mass of the gauge field and m the fermion mass. ii) For the scattering of identical fermions, the amplitude, say \mathcal{M}_p , gives a pure p-wave interaction $\mathcal{M}_p = -2(e^2/m_T^4)(m_T/m - 1)\mathbf{p} \cdot \mathbf{p}'$ where \mathbf{p} (\mathbf{p}') is the momentum of the incoming (outgoing) fermions in the center of mass frame. These imply that, for $m_T/m > 1$, s- and p-wave fermion-fermion bound states may exist. Now this argument may be doubtful since it has been given in the tree level approximation. However, since the tree level results remain unchanged at least at one-loop order [8], the arguments for the bound states still remain.

Besides on the existence of the fermion-fermion bound state pair itself, one may now consider an interesting problem of what behavior the pairs show up when they form. In ref. [2], this problem was considered for the case of identical fermions, where only the p-wave pairing is possible, and the possible vacuum instability was investigated by solving the gap equation. In this paper, we are concerned about the problem in the case where the pairing occurs in the s-wave and treat it by following the procedure of obtaining the Landau-Ginzburg description of Cooper pairs from the microscopic BCS theory [9].

The Lagrangian (1) itself would not be considered in our work. Instead of it, by mimicking the BCS theory, we consider the Lagrangian where the short-range interaction term of fermions due to photon exchange is explicitly included. The form of the interaction term can be inferred from ref. [10] where the similar situation with ours was presented. In there, the interaction being responsible for the pairing of two equally charged fermions was the four-fermion one and was represented by using the doubly charged bilinear in the fermion. The bilinear was the scalar type and given by introducing the charge conjugated fermion field, $\psi^c = C\bar{\psi}^T$, C being the charge conjugation matrix. In our case, the interaction term must be the vector type since the gauge field leads to the vector type interaction as can be shown from (1). And the fact that the s-wave pairing is that of fermions with different types in an antisymmetric manner leads us to introduce at least two types of fermions, which we label them as A type (ψ_A) and B type (ψ_B). Then the relevant interaction term can be written down as follows:

$$g(\bar{\psi}\gamma_\mu\tau^T\psi^c)(\bar{\psi}^c\gamma^\mu\tau\psi) , \quad (2)$$

where g is the coupling constant and $\psi^T = (\psi_A, \psi_B)$ is the four component spinor (ψ_A and ψ_B are two component spinors). Here the gamma matrices and τ matrix is given by

$$\gamma^0 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix} , \quad \gamma^j = \begin{pmatrix} i\sigma^j & 0 \\ 0 & i\sigma^j \end{pmatrix} , \quad \tau = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} , \quad (3)$$

$$(j = 1, 2)$$

where σ^i ($i = 1, 2, 3$) are the usual Pauli matrices and I the unit 2×2 matrix. The matrix τ plays the role of mixing A and B type fermions antisymmetrically. With these representations, the relations[†]

$$\begin{aligned}\{\gamma^\mu, \gamma^\nu\} &= 2g^{\mu\nu} , \\ \gamma^\mu \gamma^\nu &= g^{\mu\nu} - i\epsilon^{\mu\nu\rho} \gamma_\rho , \\ [\gamma^\mu, \tau] &= 0\end{aligned}\tag{4}$$

are satisfied and the charge conjugation matrix C becomes

$$C = \gamma_2 .\tag{5}$$

Now, by using the on-shell positive energy spinor [8], one can easily check that, in the nonrelativistic limit, the interaction (2) gives just the s-wave result of \mathcal{M}_s and

$$g \propto \frac{m_T}{m} - 1 ,\tag{6}$$

which means that the interaction is attractive if $g > 0$. Thus it is concluded that the interaction (2) describes correctly the interaction of fermions in the theory (1) at least in the low energy region. By the way, the coupling constant g has negative mass dimension, $[g] = -1$, and hence the four-fermion interaction (2) is not renormalizable under the conventional perturbation method. However, such interaction turns into renormalizable one in the large N perturbation theory where N is the number of fermion flavours [11]. This naturally leads us to introduce fermion flavours. Then the Lagrangian we consider is given by

$$\mathcal{L} = \bar{\psi}_i (i\not{\partial} - e\not{A} - m)\psi_i - \frac{g}{N} (\bar{\psi}_i \gamma_\mu \tau^T \psi_i^c) (\bar{\psi}_j^c \gamma^\mu \tau \psi_j)\tag{7}$$

where flavour indices i and j range from 1 to N . The gauge field A_μ is purely external and the Maxwell-CS kinetic term for it is omitted here. As an important symmetry, this Lagrangian is invariant under the $U(1)$ local gauge transformations: $\psi \rightarrow e^{i\Lambda} \psi$ and $A_\mu \rightarrow A_\mu - (1/e) \partial_\mu \Lambda$.

[†]Our conventions are $g^{\mu\nu} = \text{diag}(1, -1, -1)$, $\epsilon^{012} = +1$ and $\epsilon^{ij} = \epsilon^{0ij}$.

The four-fermion term in (7) may be linearized by introducing an auxiliary complex vector field χ_μ . The linearized form of (7) is then

$$\mathcal{L} = \bar{\psi}_i(i\cancel{\partial} - e\cancel{A} - m)\psi_i - \chi_\mu^* \bar{\psi}_i^c \gamma^\mu \tau \psi_i - \chi_\mu \bar{\psi}_i \gamma^\mu \tau^T \psi_i^c + \frac{N}{g} \chi_\mu^* \chi^\mu. \quad (8)$$

In order to maintain the gauge invariance of \mathcal{L} , the vector field should transform as $\chi_\mu \rightarrow e^{i2\Lambda} \chi_\mu$, which means that the charge of the vector field is $2e$. By analogy with the BCS theory, the vector field χ_μ in (8) is interpreted as the condensate and describes the fermion-fermion pair, which may be viewed from the equation of motion for the field, $\chi_\mu = (g/N) \bar{\psi}^c \gamma_\mu \tau \psi$. In what follows, we calculate the low energy effective action for the vector and the gauge fields, after the investigation of the phase structure according to the coupling constant g . All the formulations are performed to leading order in $1/N$ and the flavour indices are omitted from now on. As a remark, it should be emphasized that only the low energy effective action is meaningful since the four-fermion term in (7) reflects only the low energy aspect of the interaction between fermions in the system (1).

The Lagrangian (8) is bilinear in the fermion field, and may be integrated out for the field. Although it is so, some care is needed because of the presence of the charge conjugated field, ψ^c . We first recall the invariance of $\bar{\psi}(i\cancel{\partial} - e\cancel{A} - m)\psi$ under the charge conjugation;

$$\bar{\psi}^c(i\cancel{\partial} - e\cancel{A}^c - m)\psi^c = \bar{\psi}(i\cancel{\partial} - e\cancel{A} - m)\psi \quad (9)$$

where the charge conjugated gauge field A_μ^c is given by $-A_\mu$ as usual. By using (9) and introducing new field variable θ ,

$$\theta = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi \\ \psi^c \end{pmatrix}, \quad (10)$$

we can rewrite the Lagrangian (8) as follows:

$$\mathcal{L} = \bar{\theta} K \theta + \frac{N}{g} \chi_\mu^* \chi^\mu \quad (11)$$

where K is a matrix and defined by

$$K = \begin{pmatrix} i\not{\partial} - e\not{A} - m & -2\chi_\mu \gamma^\mu \tau^T \\ -2\chi_\mu^* \gamma^\mu \tau & i\not{\partial} + e\not{A} - m \end{pmatrix}. \quad (12)$$

This Lagrangian is clearly bilinear in the θ fields.

Now the effective action for the vector and gauge fields may be attained by integrating over the θ fields:

$$\exp iS_{\text{eff}}[\chi, \chi^*, A] = \int D\bar{\theta} D\theta \exp \left(i \int d^3x \mathcal{L} \right). \quad (13)$$

The formal expression of S_{eff} is

$$S_{\text{eff}} = \frac{N}{g} \int d^3x \chi_\mu^* \chi^\mu - i \frac{N}{2} \ln \det K \quad (14)$$

where \det is the functional determinant. It should be noted here that the factor $1/2$ is multiplied to the second term of the r.h.s. of eq. (14) since the θ integration gives double the result obtained from the original ψ integration as can be known from the definition of θ (10). $\det K$ in (14) may not be directly evaluated because of the matrix nature of K . To make it more tractable, we decompose K by using the following identity:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A & 0 \\ C & 1 \end{pmatrix} \begin{pmatrix} 1 & A^{-1}B \\ 0 & D - CA^{-1}B \end{pmatrix}. \quad (15)$$

We then obtain, after a few manipulations,

$$\det K = \det^2(i\not{\partial} - e\not{A} - m) \det \left(1 - 4 \frac{1}{i\not{\partial} + e\not{A} - m} \chi^* \cdot \gamma \frac{1}{i\not{\partial} - e\not{A} - m} \chi \cdot \gamma \right), \quad (16)$$

where we use the fact that $\det(i\not{\partial} + e\not{A} - m)$ is equal to $\det(i\not{\partial} - e\not{A} - m)$ due to Furry's theorem and remove the τ matrices by using eq. (4). By substituting this into eq. (14), the effective action becomes

$$S_{\text{eff}} = S_A + S_{\chi A} \quad (17)$$

where

$$S_A = -iN \text{Tr} \ln(i\not{\partial} - e\not{A} - m), \quad (18)$$

$$S_{\chi A} = \frac{N}{g} \int d^3x \chi_\mu^* \chi^\mu - i \frac{N}{2} \text{Tr} \ln \left(1 - 4 \frac{1}{i \not{\partial} + e \not{A} - m} \chi^* \cdot \gamma \frac{1}{i \not{\partial} - e \not{A} - m} \chi \cdot \gamma \right) . \quad (19)$$

Here, we used the identity $\ln \det X = \text{Tr} \ln X$ where Tr is the functional trace. If we set the field χ_μ equal to zero, the effective action reduces to the usual fermion determinant, S_A , as it should be.

Before we calculate the effective action, we first concentrate on the phase structure in terms of the coupling constant g . The phase structure is obtained by investigating the saddle point or gap equation involving the vacuum expectation value v of the field χ_μ . Here, v is taken as the Lorentz invariant scalar, and given by

$$v = \langle |\chi| \rangle , \quad (20)$$

with $|\chi| = (\chi_\mu^* \chi^\mu)^{1/2}$. For the gap equation, we need to calculate the effective potential V_{eff} , which is defined as $\int d^3x V_{\text{eff}} = -S_{\text{eff}}$ where the value of χ_μ is constant and the gauge field is turned off:

$$V_{\text{eff}} = -\frac{N}{g} \chi_\mu^* \chi^\mu + i \frac{N}{2} \int^\Lambda \frac{d^3p}{(2\pi)^3} \text{tr} \ln \left(1 - 4 \frac{1}{\not{p} - m} \gamma^\mu \frac{1}{\not{p} - m} \gamma^\nu \chi_\mu^* \chi_\nu \right) , \quad (21)$$

with Λ the spatial momentum cutoff and tr the trace of gamma matrices. This potential, however, is not computed easily because the structure of the gamma matrices in the logarithmic function of eq. (21) is not simplified. Here, we restrict ourselves to the case where the value of χ_μ is close to zero. In this case, V_{eff} may be expanded in powers of χ_μ and hence the gap equation may be obtained approximately. If we expand V_{eff} up to the first nontrivial order, then the gap equation becomes, after a straightforward calculation,

$$\begin{aligned} 0 &= \frac{1}{N} \frac{\delta V_{\text{eff}}}{\delta v} \\ &\simeq 2v \left(-\frac{1}{g} + \frac{2}{3\pi} \sqrt{\Lambda^2 + m^2} + \frac{8v^2}{9\pi|m|} \right) . \end{aligned} \quad (22)$$

The solution of this equation with nonvanishing v (> 0) exists only when g is smaller than the critical coupling $g_c^{-1} \equiv (2/3\pi) \sqrt{\Lambda^2 + m^2}$. In that region, v is given by

$$v^2 \simeq \frac{9\pi|m|}{8} \left(\frac{1}{g} - \frac{1}{g_c} \right) , \quad 0 < g < g_c . \quad (23)$$

This clearly shows that the condensation of fermion-fermion pairs occurs in the weak coupling phase. This result is the reasonable one if we recall the previous discussion on the nonrelativistic limit of the interaction (2), in particular eq. (6) ($g > 0$ corresponds to the attraction between fermions). In refs. [2,7], it has been argued that, if m_T becomes very large so that $m_T \gg m$, the fermion-fermion bound states would not form in the theory (1). This is further supported by the weak coupling result (23), since, according to eq. (6), the region of $m_T \gg m$ corresponds to the strong coupling phase where the condensation does not appear. Now we would like to note that eq. (23) is valid only in the region near the critical point; if g is far from g_c , eq. (23) cannot be trusted.

We now turn our interest to the effective action S_{eff} . For its evaluation, we take the derivative expansion method [12], which is well suited for obtaining the low energy effective action. The fermion determinant S_A would not be evaluated, since it is well known that its evaluation leads just to the CS action in the low energy limit [13]. In particular, the evaluation using the derivative expansion was also given in ref. [14] where, of course, the same result was reproduced. Thus we would only quote the result which is given by

$$\frac{1}{N}S_A \simeq -\frac{e^2}{4\pi} \frac{m}{|m|} \int d^3x \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda . \quad (24)$$

If we recover the original CS action for the gauge field which was omitted in the Lagrangian (7), this action leads just up to the renormalization of the CS coefficient κ , and the renormalized coefficient, κ_r , becomes $\kappa_r = \kappa + \frac{Ne^2m}{|m|}$. We evaluate the $S_{\chi A}$ up to quadratic order in the field χ_μ in the low energy limit and near the critical point g_c , where the vacuum expectation value v is very close to zero and hence the lowest order contribution in v is dominant. The primary contribution comes from the term containing single derivative, $\chi^* \partial \chi$. Since the vector field χ_μ transforms under the gauge transformation, this term is not gauge invariant. To make it gauge invariant, we must consider the coupling term with the gauge field. The coupling terms may be obtained by expanding the denominators of eq. (19) in powers of the gauge field. The first one, which is needed here, is of the form of $A\chi^*\chi$. The non derivative quadratic term is $\chi^*\chi$ and may be inferred from the gap equation (22). If

we assemble the calculations for the three terms until now, we get the expression of S_{χ_A} as follows:

$$\frac{1}{N}S_{\chi_A} \simeq \frac{m}{\pi|m|} \int d^3x \chi_\mu^* \left[-\epsilon^{\mu\nu\lambda}(\partial_\nu + 2ieA_\nu) + g^{\mu\lambda}M \right] \chi_\lambda \quad (25)$$

with $M \equiv \pi \frac{m}{|m|} \left(\frac{1}{g} - \frac{1}{g_c} \right)$. This gives the Landau-Ginzburg like description for the s-wave fermion-fermion pairs existing in system (1) in the low energy and the critical region. If we recall the transformation of the field χ_μ under the gauge transformation, this action is clearly gauge invariant.

The behavior of the field χ_μ may now be read off from the effective action (25). Specifically, we look at the equation of motion for the field derived as follows:

$$[-\epsilon^{\mu\nu\lambda}(\partial_\nu + 2ieA_\nu) + g^{\mu\lambda}M]\chi_\lambda = 0 . \quad (26)$$

If we ignore the gauge field for a moment, this is just the equation of motion for the spin 1 field following from the representation theory of the Poincaré algebra in 2+1 dimensions [15,16]. Thus the quanta of the field χ_μ behave like spin 1 vector bosons with the charge of amount $2e$. However, it should be noted that this is true only in the case of the weak coupling phase. At the critical point, $g = g_c$, the mass term for the field χ_μ in (25) disappears and hence the field χ_μ becomes massless. Since the massless field is spinless in 2+1 dimensions [1], the field χ_μ is so in this case, and it is explicitly distinguished from that in the weak coupling case. Above the critical point, that is, in the strong coupling phase, $g > g_c$, there is no condensation of χ_μ . This implies that the fermion-fermion pairs do not form and hence it is meaningless to consider the behavior of the field χ_μ describing them.

It is not so difficult to understand the fact that χ_μ is the spin 1 field in the weak coupling phase. In 2+1 dimensions, the massive Dirac fermion has only one spin degree of freedom; the spin of it can take only one of two possible values $\pm\frac{1}{2}$ [1,17]. If we specify $m > 0$, the spin of the fermion (antifermion) is $+\frac{1}{2}$ ($-\frac{1}{2}$). Thus the spin of the fermion-fermion pair is simply given by adding spin values of two constituent fermions, and becomes $+1$. This contrasts with that of Cooper pairs in the usual BCS theory: the Cooper pair is constituted

by two electrons with spin up and down respectively, and is described by spin zero scalar field.

Now we would like to address the question whether or not the Meissner effect takes place in the weak coupling phase. This may be answered by investigating the appearance of the longitudinal mass of the gauge field. The related term in the derivative expansion of (19) is $A^2\chi^*\chi$. The direct calculation for it yields the result as follows:

$$N \frac{8e^2}{\pi|m|} \int d^3x A_\mu A^\mu \chi_\nu^* \chi^\nu, \quad (27)$$

which is to be considered as one term in the effective action. This clearly shows that, if condensation occurs, the gauge field acquires the longitudinal mass depending on the value $16Ne^2v^2/\pi|m|$ or $18Ne^2mM/\pi|m|$. The longitudinal mass is the origin of the screening of external magnetic field, i.e. Meissner effect. Thus it may be concluded that the Meissner effect occurs in the weak coupling phase.

There is the similarity between the field χ_μ and the gauge field that they are all massive vector fields in the weak coupling phase. This causes us to expect the other symmetry structure in addition to the $U(1)$ gauge symmetry. In ref. [7], it was speculated that fermion-fermion and antifermion-antifermion pairs and the gauge field would form an $SU(2)$ isospin triplet state in a certain situation, but without any detail analysis. We now give an attempt for the realization of the symmetry with the low energy effective Lagrangian. Note that the symmetry is approximate one unlike the $U(1)$ gauge symmetry, since it would be valid only in the low energy limit. If we choose $m > 0$ for certainty, the effective Lagrangian, $\int d^3x \mathcal{L}_{\text{eff}} = S_{\text{eff}}$, is written as

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \frac{N}{\pi} \chi_\mu^* \left[-\epsilon^{\mu\nu\lambda} (\partial_\nu + 2ieA_\nu) + g^{\mu\lambda} M \right] \chi_\lambda \\ & - \frac{\kappa_r}{4\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + \frac{N}{\pi} 9e^2 M A_\mu A^\mu + O(\partial^2, v^4) \end{aligned} \quad (28)$$

where the original CS term omitted in (7) is recovered. Although the Lagrangian seems to be gauge noninvariant, the gauge symmetry of it is guaranteed if the higher derivative terms and the higher polynomials in fields are considered. We first set

$$\chi_\mu = B_\mu^1 + iB_\mu^2, \quad A_\mu = 2\sqrt{\frac{N}{\kappa_r}}B_\mu^3 \quad (29)$$

and substitute these into (28). Then we obtain

$$\mathcal{L}_{\text{eff}} = -\frac{N}{\pi}\epsilon^{\mu\nu\lambda} \left(B_\mu^a \partial_\nu B_\lambda^a + \frac{8e\sqrt{N/\kappa_r}}{3}\epsilon^{abc} B_\mu^a B_\nu^b B_\lambda^c \right) + \frac{N}{\pi} M^{ab} B_\mu^a B^{b\mu} + O(\partial^2, v^4) \quad (30)$$

where the isospin indices a , b , and c take the values of 1, 2 and 3 and the mass matrix M^{ab} is given by

$$M^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{36e^2 N}{\kappa_r} \end{pmatrix} M. \quad (31)$$

Now, if κ_r takes the special value of $36e^2 N$, the Lagrangian (30) is invariant under the global $SU(2)$ isospin transformation,

$$B_\mu^a \rightarrow U^{ab}(\phi) B_\mu^b, \quad (32)$$

where the matrix $U(\phi) = \exp(i\phi^a T^a)$ is the element of the $SU(2)$ group in the adjoint representation with the generators $(T^a)^{bc} = -i\epsilon^{abc}$. Thus, in this special case, the field χ_μ , its conjugate field and the gauge field form an $SU(2)$ isospin triplet in the long wave-length limit and in the weak coupling phase. At the critical point, the mass matrix M^{ab} vanishes, and the Lagrangian becomes just the non-Abelian CS theory which enjoys the local $SU(2)$ symmetry. This yields an interesting conclusion that the low energy nature of the system would become topological at the critical point.

In summary, we have been concerned about the s-wave fermion-fermion pairs which may appear in the topologically massive QED₂₊₁ with some generalization. By considering the Lagrangian where the appropriate four-fermion term responsible for the pairing is explicitly included, we have given the Landau-Ginzburg like description for them in the low energy limit and near the critical point. It has been shown that they are described by doubly charged spin 1 vector bosons and, when they condense, the gauge field acquires the longitudinal mass indicating the Meissner effect. Provided that $\kappa_r = 36e^2 N$, it has been realized that there

appears the global $SU(2)$ isospin symmetry due to the similarity between the condensate fields and the gauge field in the condensation phase.

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